Poynting Theory & Wave Polarization

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Agenda

- Poynting Theory
  - Poynting Vector
  - Time average power density
- Polarization of plane waves
  - Linear polarization
  - Circular polarization (RHCP &LHCP)
  - Elliptical polarization (RHEP &LHEP)

Poynting's Theorem

- A propagating (EM) wave carries energy with it. Physically this makes sense to us when we listen to the radio or talk on a cell phone. These types of wireless communications are possible because EM waves carry energy.
- In these examples, some of this EM energy is used to oscillate electrons in the metal parts of the receiving antenna of our radio or cell phone, which ultimately results in wireless communications.

Derivation of Poynting's Theorem

- Poynting's theorem concerns the conservation of energy for a given volume in space.
- Poynting's theorem is a consequence of Maxwell's equations.
- It is a hugely important mathematical statement in electromagnetics that concerns the flow of power through space. We will derive it now from time-domain fields.

Poynting vector

Poynting Vector “S” is defined as the cross product of the vectors E & H.

The direction of power flow at any point is normal to both E & H vectors.

The Unit is watts/m^2

$$S = E \times H$$

Poynting Theorem from Maxwell’s Equations:

Maxwell’s equation in the point form is

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Equation (1)

Taking dot product with “E” on both sides

$$E \cdot (\nabla \times H) = E \cdot J + E \cdot \frac{\partial D}{\partial t}$$

From vector identity,

$$\nabla \cdot (E \times H) = H \cdot (\nabla \times E) - E(\nabla \times H)$$

Equation (2)

$$E \cdot (\nabla \times H) = H \cdot (\nabla \times E) - \nabla \cdot (E \times H)$$

Substituting equation (2) in (1)
Equation (3) is

$$H \cdot (\nabla \times E) - \nabla \cdot (E \times H) = E \cdot J + E \cdot \frac{\partial D}{\partial t}$$

From Maxwell's Equation $(\nabla \times E) = -\frac{\partial B}{\partial t}$

$$H\left(\frac{-\partial B}{\partial t}\right) - \nabla \cdot (E \times H) = E \cdot J + E \cdot \frac{\partial D}{\partial t}$$

Equation (4)

$$\nabla \cdot (E \times H) = -E \cdot J - H \cdot \left(\frac{\partial B}{\partial t}\right) - E \cdot \frac{\partial D}{\partial t}$$

Substituting $J = \sigma E, D = \varepsilon E\ , B = \mu H$

in Equation (4)

$$\nabla \cdot (E \times H) = -\sigma E^2 - E \cdot \frac{\partial(\varepsilon E)}{\partial t} - H\frac{\partial(\mu H)}{\partial t}$$

$$\nabla \cdot (E \times H) = -\sigma E^2 - \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon E^2\right) - \frac{\partial}{\partial t}(\frac{1}{2}\mu H^2)$$

Integrating throughout the volume

$$\int_{vol} \nabla \cdot (E \times H) dv = -\int_{vol} \sigma E^2 dv - \int_{vol} \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right) dv$$

Total Power | Power dissipation | Rate of energy stored

Using divergence theorem

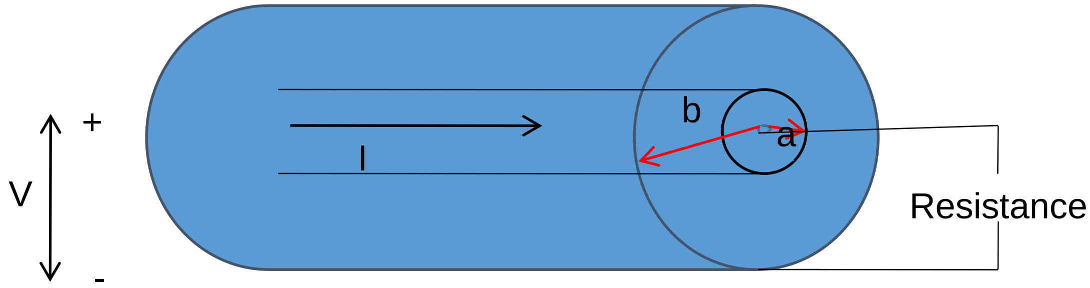
$$\int_S D.ds = \int_{vol} \nabla \cdot D \, dv$$

$$\int_{vol} \nabla \cdot (E \times H) dv = \int_S (E \times H) \cdot dS$$

Power flow in a co-axial cable

Consider a co-axial cable which has a dc voltage ‘V’ between the conductors and a steady current I flowing in the inner and outer conductors.

The radius of inner and outer conductor are ‘a’ and ‘b’ respectively.



By ampere's Law:

$$\int H \cdot dL = I$$

$$\int dL =$$ Circumference of circular path between a and b =2πr

$$H \cdot (2\pi r) = I$$

$$H = \frac{I}{2\pi r} \quad a < r < b$$

E due to an infinitely long conductor

$$E = \frac{\lambda}{2\pi\varepsilon r} \qquad \text{Equation (1)}$$

Where λ is the charge density.

The potential difference between the conductors is

$$V = \frac{\lambda}{2\pi\epsilon} In\left(\frac{b}{a}\right) \qquad \text{Equation (2)}$$

E in terms of V from (1) and (2) is

$$E = \frac{V}{In\left(\frac{b}{a}\right)r}$$

Power density $P = E \times H$

Since E and H are always perpendicular to each other

$$P = E \cdot H$$

$$P = \frac{V}{In\left(\frac{b}{a}\right)r} \cdot \frac{I}{2\pi r}$$

The total power will be given by the integration of power density P over any cross section surface.

Let the elemental surface already be 2πrdr

Total power

$$W = \int \frac{V}{In\left(\frac{b}{a}\right) r} \cdot \frac{I}{2\pi r} (2\pi r)\, dr$$

$$W = \frac{V}{In\left(\frac{b}{a}\right)} I \int_a^b \frac{1}{r}\, dr$$

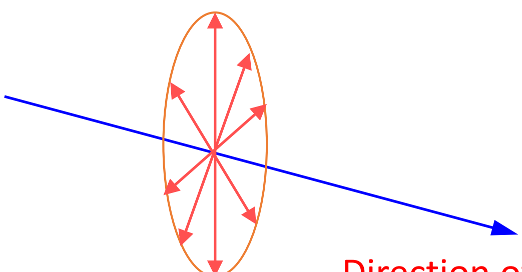
$$\boxed{W = \frac{V}{In\left(\frac{b}{a}\right)} \cdot I\left(In\frac{b}{a}\right)}$$

$$\boxed{W = VI}$$

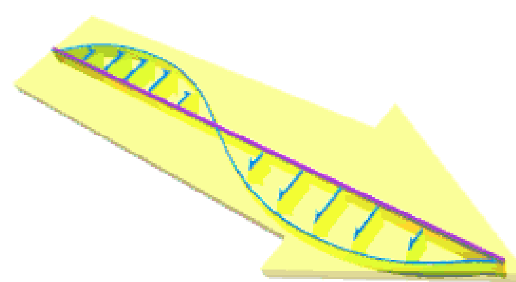
i.e. The power flow along the cable is the product of V and I

Polarization

- **Polarization** is a property applying to transverse waves that specifies the geometrical orientation of the oscillations.
- Oscillation which take places in a transverse wave in many different directions is said to be unpolarized.
- In an unpolarized transverse wave oscillations may take place in any direction at right angles to the direction in which the wave travels.

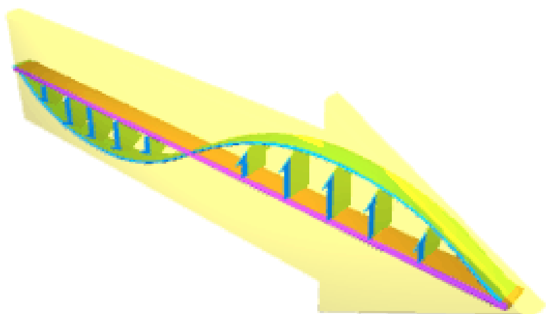


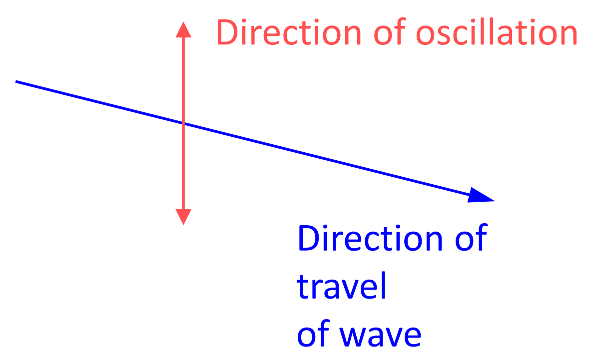
Direction of
propagation
of wave



Linear Polarization

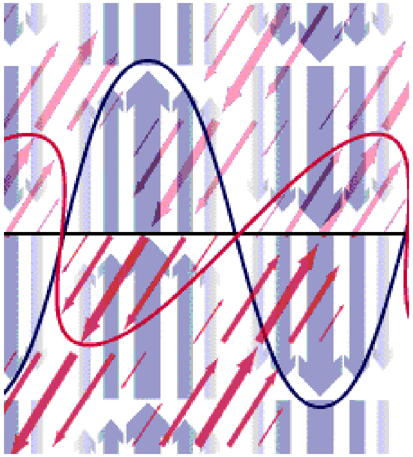
- If the oscillation does take place in only one direction then the wave is said to be linearly polarized (or plane polarized) in that direction.





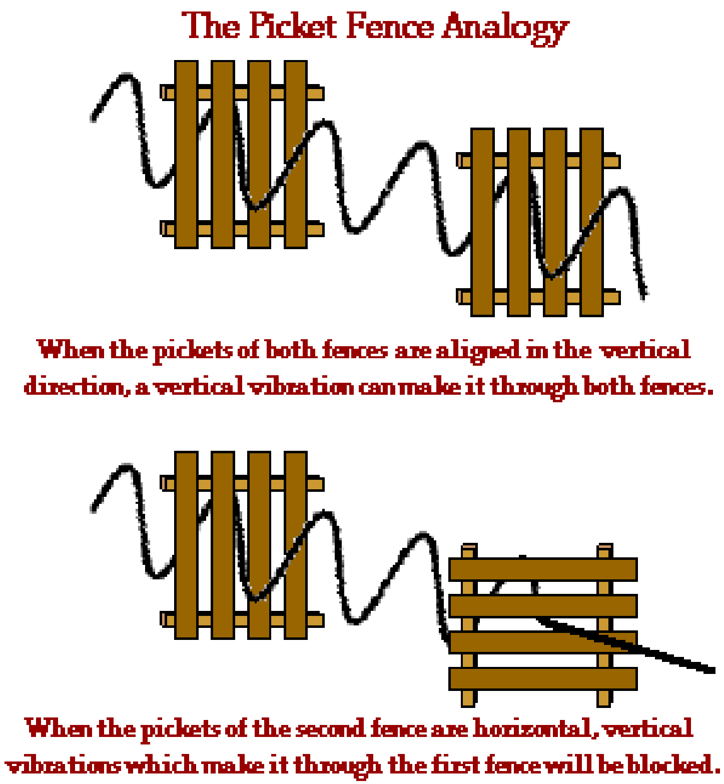
Polarization of Electromagnetic Waves

- Any electromagnetic wave consists of an electric field component and a magnetic field component.
- The electric field component is used to define the plane of polarization because many common electromagnetic-wave detectors respond to the electric forces on electrons in materials, not the magnetic forces.



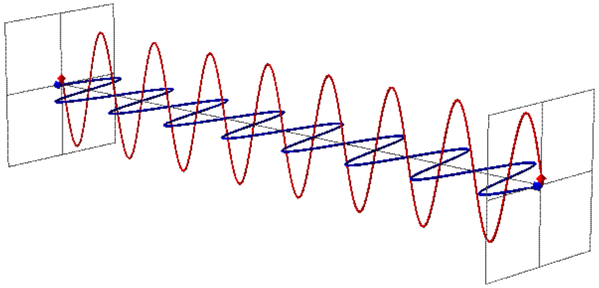
Polarization by Selective Absorption

- Polarization of wave by selective absorption is analogous to that shown in the diagrams.

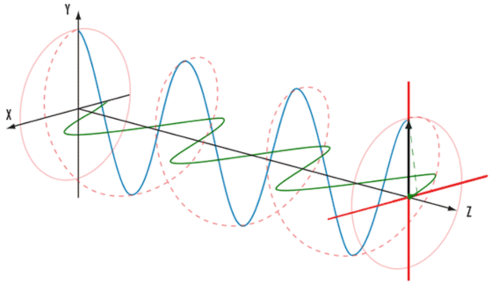


Wave Polarization

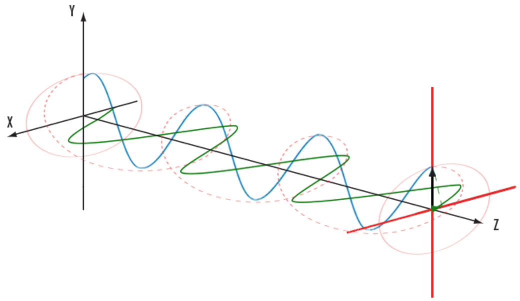
- We define the *wave polarization* as the time-dependent electric field vector orientation at a fixed point in space.
- Or the locus of the end point of the electric field vector at a fixed plane in space.



Linear



Circular



Elliptical

Linear polarization

$$\overline{E}(z,t) = E_0 \cos(\omega t - \beta z)\hat{a}_x$$

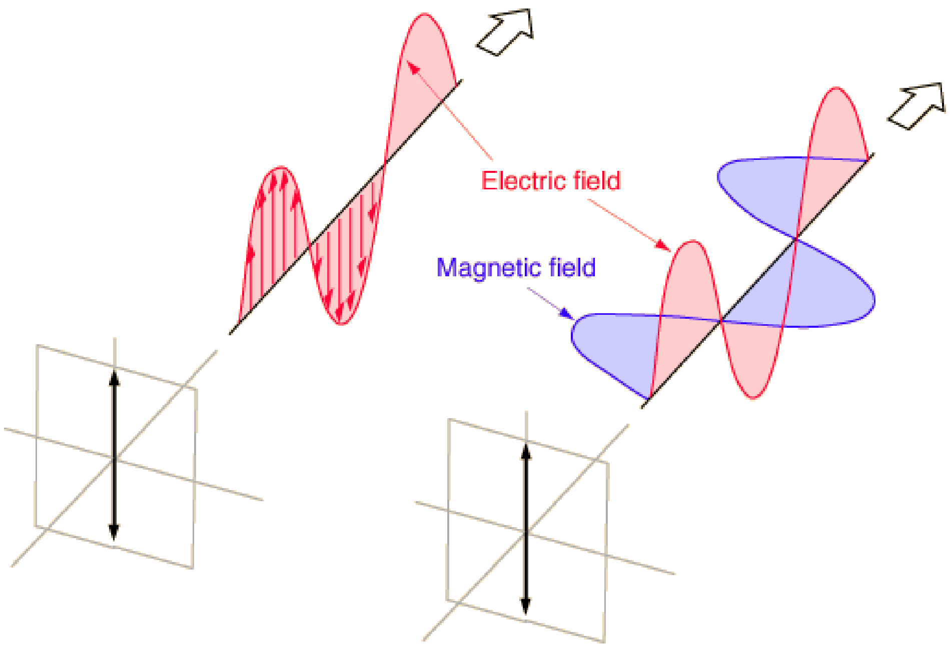
At z = 0

$$\overline{E}(z,t) = E_0 \cos(\omega t)\hat{a}_x$$

At ωt = 0 ➡ $\overline{E} = E_0 \hat{a}_x$

At $\omega t = \frac{\pi}{4}$ ➡ $\overline{E} = \frac{E_0}{\sqrt{2}} \hat{a}_x$

At $\omega t = \frac{\pi}{2}$ ➡ $\overline{E} = 0\hat{a}_x$



Linear polarization

For general if the electric field vector is composed of two components

$$\overline{E}(z,t) = E_1 \cos(\omega t - \beta z)\hat{a}_x + E_2 \cos(\omega t - \beta z + \phi)\hat{a}_y$$

Case 1: $\Phi = 0,\ E_1 = E_2$

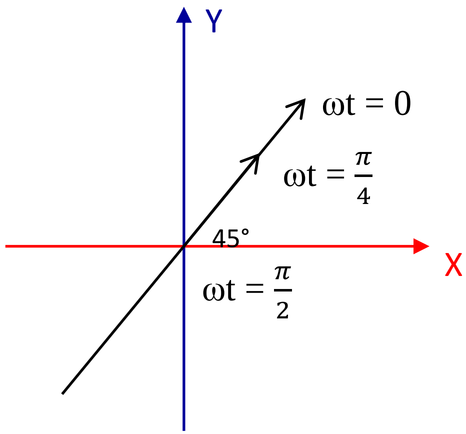
At z = 0

$$\overline{E}(z,t) = E_1 \cos(\omega t)[\hat{a}_x + \hat{a}_y]$$

At $\omega t = 0$ $\Rightarrow$ $\overline{E} = E_1[\hat{a}_x + \hat{a}_y]$

At $\omega t = \frac{\pi}{4}$ $\Rightarrow$ $\overline{E} = \frac{E_1}{\sqrt{2}}[\hat{a}_x + \hat{a}_y]$

At $\omega t = \frac{\pi}{2}$ $\Rightarrow$ $\overline{E} = 0[\hat{a}_x + \hat{a}_y]$



Linear polarization

For general if the electric field vector is composed of two components

$$\overline{E}(z,t) = E_1 \cos(\omega t - \beta z)\hat{a}_x + E_2 \cos(\omega t - \beta z + \phi)\hat{a}_y$$

Case 1: $\Phi = 0,\ E_1 \neq E_2$

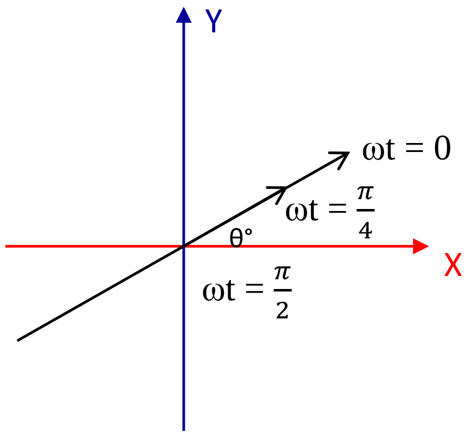
At z = 0

$$\overline{E}(z,t) = E_1 \cos(\omega t)\hat{a}_x + E_2 \cos(\omega t)\hat{a}_y$$

At $\omega t = 0$ ⇨ $\overline{E} = E_1\hat{a}_x + E_2\hat{a}_y$

At $\omega t = \frac{\pi}{4}$ ⇨ $\overline{E} = \frac{E_1}{\sqrt{2}}\hat{a}_x + \frac{E_1}{\sqrt{2}}\hat{a}_y$

At $\omega t = \frac{\pi}{2}$ ⇨ $\overline{E} = 0\hat{a}_x + 0\hat{a}_y$

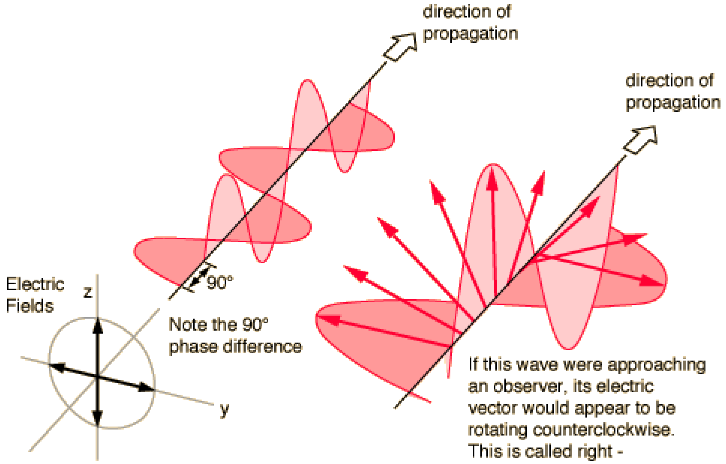


$$\theta = \tan^{-1}\frac{E_2}{E_1}$$

Circular polarization

circular polarization of an electromagnetic wave is a polarization in which the electric field of the passing wave does not change strength but only changes direction in a rotary manner.

Circularly polarized wave consists of two perpendicular electromagnetic plane waves of equal amplitude and 90° difference in phase.



Circular polarization

$$\overline{E}(z,t) = E_1 \cos(\omega t - \beta z)\hat{a}_x + E_2 \cos(\omega t - \beta z + \phi)\hat{a}_y$$

Case 2: $\Phi = -\frac{\pi}{2}$, $E_1 = E_2$

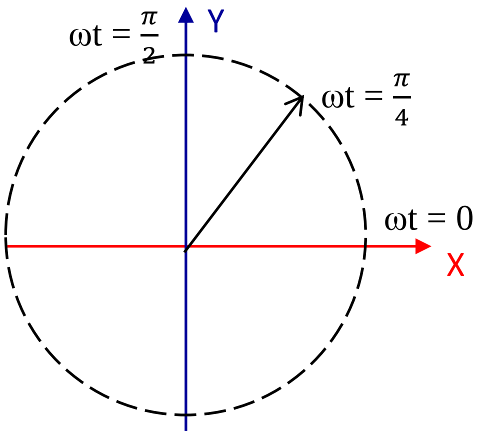
At z = 0

$$\overline{E}(z,t) = E_1 \cos(\omega t)\hat{a}_x + E_1 \cos(\omega t - \frac{\pi}{2})\hat{a}_y$$

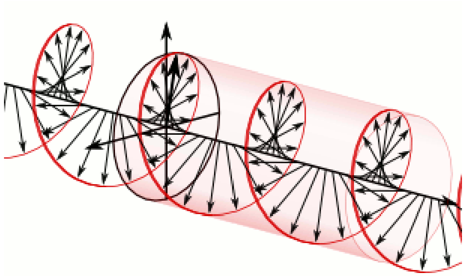
At ωt = 0 ➡ $\overline{E} = E_1\hat{a}_x$

At $\omega t = \frac{\pi}{4}$ ➡ $\overline{E} = \frac{E_1}{\sqrt{2}}\hat{a}_x + \frac{E_1}{\sqrt{2}}\hat{a}_y$

At $\omega t = \frac{\pi}{2}$ ➡ $\overline{E} = E_1\hat{a}_y$



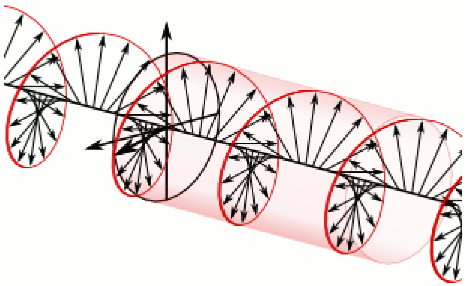
Circular polarization



LHCP

A left-handed/anti-clockwise circularly polarized wave as defined from the point of view of the source.

$$\Phi = \frac{\pi}{2}$$



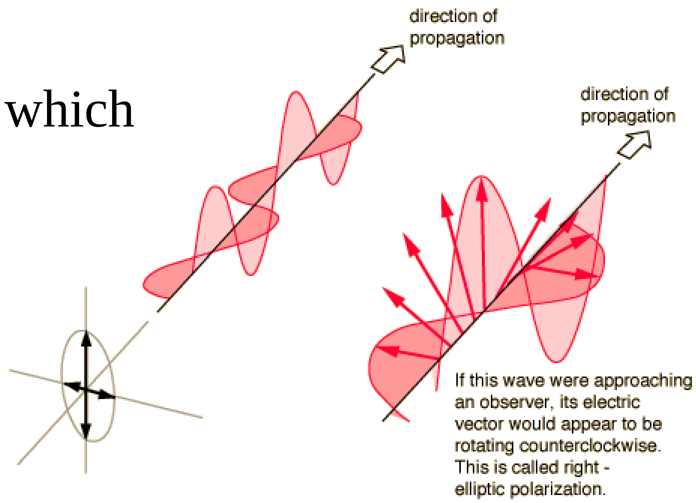
RHCP

A right-handed/clockwise circularly polarized wave as defined from the point of view of the source.

$$\Phi = -\frac{\pi}{2}$$

Elliptical polarization

Elliptically polarized wave consists of two perpendicular waves of unequal amplitude which differ in phase by 90°.



Elliptical polarization

$$\overline{E}(z,t) = E_1\cos(\omega t - \beta z)\hat{a}_x + E_2\cos(\omega t - \beta z + \phi)\hat{a}_y$$

Case 2: $\Phi = -\frac{\pi}{2}$, $E_1 \neq E_2$

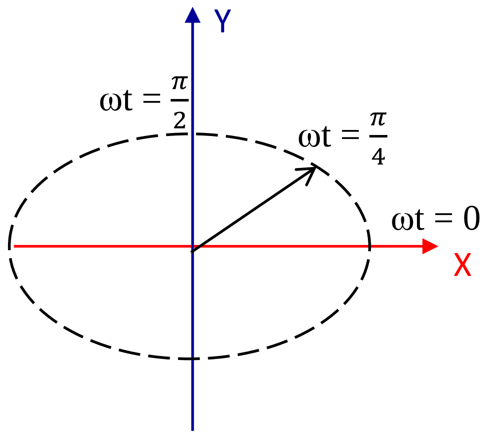
At z = 0

$$\overline{E}(z,t) = E_1\cos(\omega t)\hat{a}_x + E_2\cos(\omega t - \frac{\pi}{2})\hat{a}_y$$

At ωt = 0 ⇨ $\overline{E} = E_1\hat{a}_x$

At $\omega t = \frac{\pi}{4}$ ⇨ $\overline{E} = \frac{E_1}{\sqrt{2}}\hat{a}_x + \frac{E_2}{\sqrt{2}}\hat{a}_y$

At $\omega t = \frac{\pi}{2}$ ⇨ $\overline{E} = E_2\hat{a}_y$



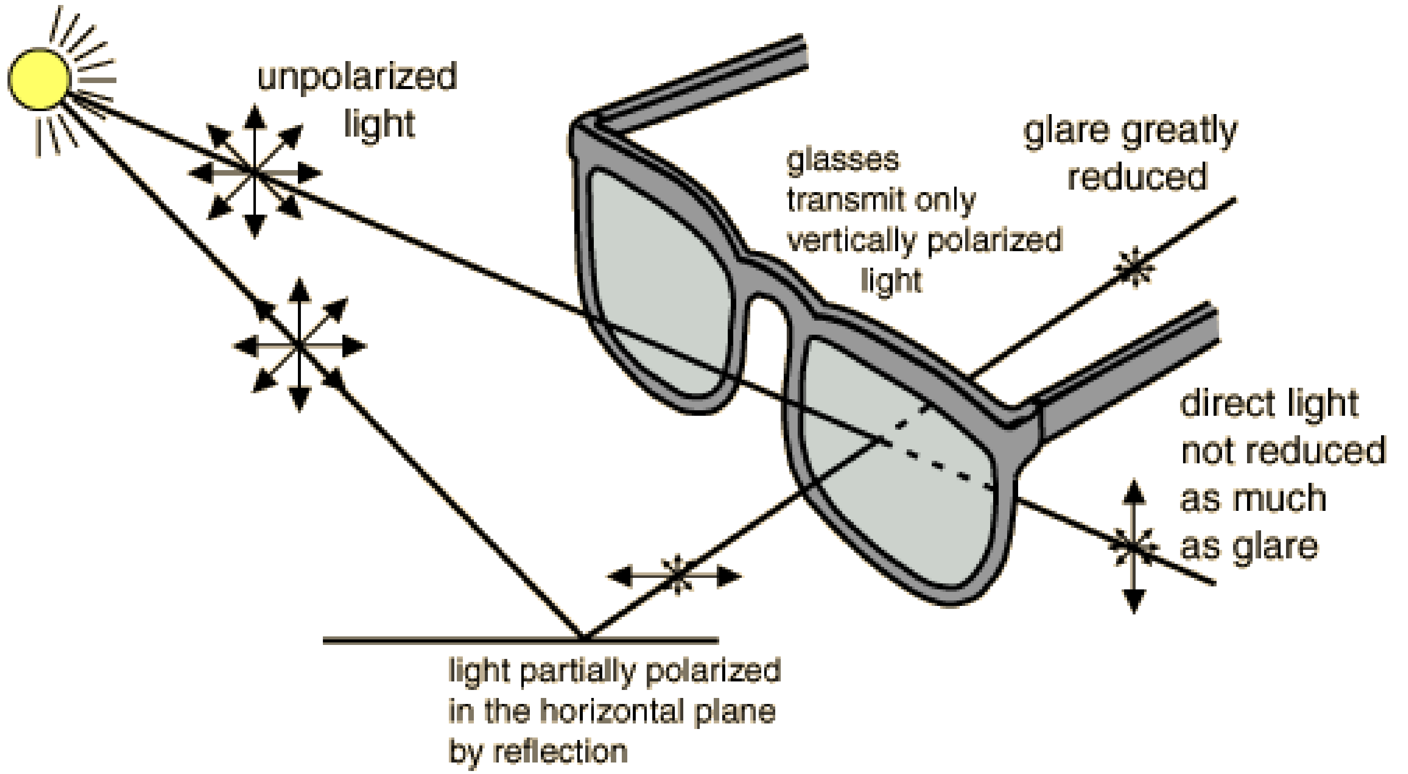
Polarization applications

Polarized sun glasses

- Polarized lenses are those that have been treated in a way that reduces glare.
- polarized lenses are designed to absorb all light waves coming from directions that are not vertical. That is, horizontal light waves are absorbed, and this prevents glare, which usually interferes with the quality of view.
- Polarized lenses do not just help you avoid glare, they block harmful UV rays and this protects your eyes.

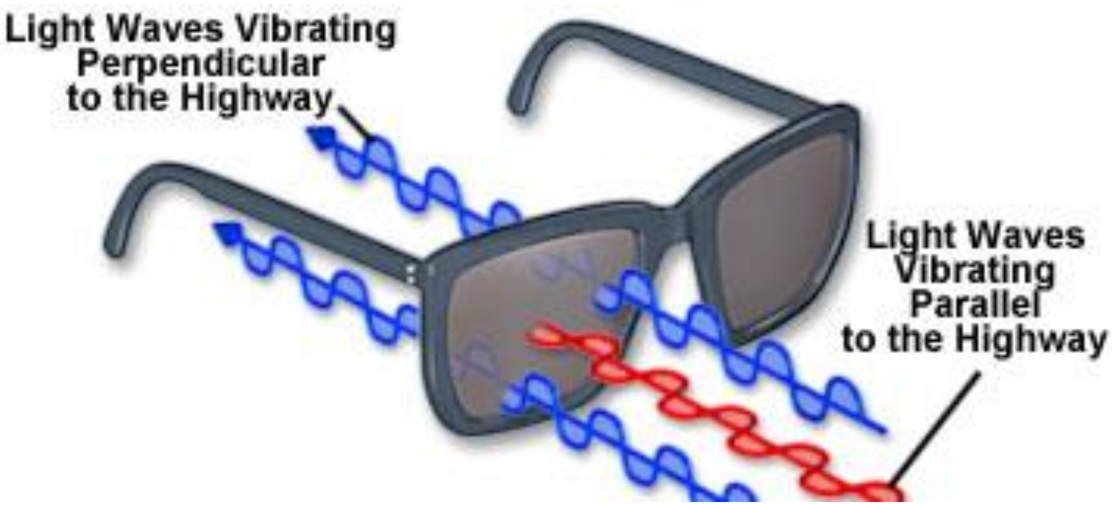
Polarization applications

Polarized sun glasses



Polarization applications

Polarized sun glasses



Polarization applications

- Polarization is also used to produce and show 3-D movies.
- Three-dimensional movies are actually two movies being shown at the same time through two projectors.
- The two movies are filmed from two slightly different camera locations. Each individual movie is then projected from different sides of the audience onto a metal screen.
- The movies are projected through a polarizing filter. The polarizing filter used for the projector on the left may have its polarization axis aligned horizontally while the polarizing filter used for the projector on the right would have its polarization axis aligned vertically. Consequently, there are two slightly different movies being projected onto a screen. Each movie is cast by light that is polarized with an orientation perpendicular to the other movie. The audience then wears glasses that have two Polaroid filters. Each filter has a different polarization axis - one is horizontal and the other is vertical. The result of this arrangement of projectors and filters is that the left eye sees the movie that is projected from the right projector while the right eye sees the movie that is projected from the left projector. This gives the viewer a perception of depth.